Raman Cooling of Cesium below 3 nK: New Approach Inspired by Lévy Flight Statistics

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We present a new approach to optimize subrecoil Raman cooling, based on Lévy flight statistics. It shows that simple time sequences using square pulses can lead to very efficient cooling. We tested the method in a one-dimensional experiment with cesium atoms and obtained temperatures below 3 nK, less than 1/70 of the single photon recoil temperature.

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In laser cooling of atoms, the natural velocity scale is the recoil velocity $v_R = \hbar k/M$ of an atom with mass M absorbing or emitting a single photon with momentum $\hbar k$. Most laser cooling methods lead to velocity spreads δv of a few v_R. Because of the random nature of spontaneous emission which occurs during cooling, it is not easy to achieve $\delta v < v_R$, i.e., to enter the subrecoil regime where the effective temperature T_{eff} defined by $k_B T_{\text{eff}}/2 = M(\delta v)^2/2$ is lower than the recoil temperature T_R defined by $k_B T_R/2 = (\hbar k)^2/2M$. To date, two subrecoil cooling methods have been demonstrated on free atoms: velocity selective coherent population trapping (VSCPT) [1] and Raman cooling [2]. Both methods use a combination of two effects: (i) a vanishing absorption rate of light for atoms around $v = 0$, which protects them from the random recoil induced by spontaneous emission, and (ii) a drift and diffusion of atoms in velocity space bringing the atoms from the $v \neq 0$ domain, where they scatter light, into the $v \simeq 0$ region, where they do not absorb light, remain trapped, and accumulate. VSCPT has reached in 1D $T_R/20$ [3] then $T_R/40$ [4], in 2D $T_R/16$ [5], and very recently $T_R/22$ in 3D [6], while Raman cooling has reached $T_R/20$ in 1D [2] and $0.75T_R$ in 2D [7].

In subrecoil cooling, one expects the temperature T_{eff} to decrease with the interaction time Θ. For the case of VSCPT, theory predicts $T_{\text{eff}} \propto \Theta^{-1}$ [8], while for Raman cooling no quantitative predictions have been given so far. In an actual experiment, Θ is always limited for practical reasons. The following question then arises: Given Θ, what is the best strategy for cooling a maximum number of atoms to the lowest possible temperature? We address this question in this paper for the case of Raman cooling, using as a guide simple arguments provided by Lévy flights. In this way we have been able to cool cesium atoms in one dimension below 3 nK ($T_R/70$) and to reach a quantitative understanding of the cooling process, allowing one to derive analytical expressions for the optimum cooling parameters.

In the Lévy-flight approach [9], one defines a "trapping" zone around $v = 0$ by $|v| < v_{\text{trap}}$, where $v_{\text{trap}} < v_R$. The time evolution of the atom appears as a sequence of trapping periods ($|v| \leq v_{\text{trap}}$) with durations $\tau_1, \tau_2, \ldots$ alternating with "escape" periods ($|v| > v_{\text{trap}}$) with durations $\hat{\tau}_1, \hat{\tau}_2, \ldots$ (also called first return times). The cooling efficiency is determined by a competition between these two processes. Consider first the trapping periods. Their statistical properties are essentially determined by the v dependence of the absorption rate $\Gamma'(v)$ around $v = 0$, which can be generally written as

$$\Gamma'(v) \simeq \frac{1}{\tau_0} \left| \frac{v}{v_0} \right|^{\alpha} . \qquad (1)$$

The rate $\Gamma'(v)$ presents a dip centered around $v = 0$, with a characteristic width v_0 (we suppose here that $v_{\text{trap}} < v_0$). Outside the dip, $\Gamma'(v)$ has an order of magnitude given by $1/\tau_0$, with $\tau_0 \ll \Theta$ (Fig. 1). As shown in [4,9], the distribution $P(\tau)$ of trapping times is controlled by the exponent α of Eq. (1). For large τ, and in one dimension, one finds that $P(\tau) \simeq B\tau^{-(1+1/\alpha)}$, where B is a constant prefactor depending on v_0, τ_0, and v_{trap}. Such a power law dependence of $P(\tau)$ is a clear signature of the appearance of Lévy statistics in the problem [4,9,10]. The total trapping time $T(N) = \sum_{i=1}^{N} \tau_i$, where N is the number of trapping events during Θ, obeys the generalized central limit theorem of Lévy and Gnedenko [10]. In particular, if $\alpha \geq 1$, $P(\tau)$ is a

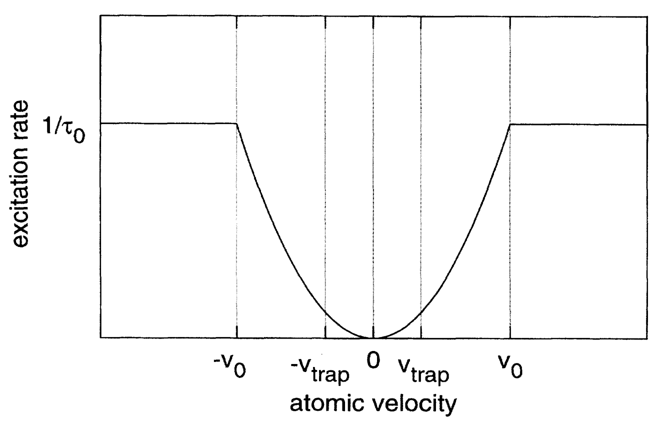

FIG. 1. The velocity dependent excitation rate is modeled by the function $\Gamma'(v) \propto |v|^{\alpha}$ around $v = 0$ and assumed constant for $v > v_0$.

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"broad" distribution which, unlike common distributions, decays so slowly that $\langle\tau\rangle$ is infinite. In such a case, the usual relation $T(N) \simeq N\langle\tau\rangle$ (for $N \gg 1$) no longer holds and one can show that $T(N)$ grows more rapidly, as N^α (or as $N \log N$ if $\alpha = 1$). On the other hand, since Raman cooling provides an effective friction mechanism [2], atoms tend to return rapidly into the trap. The distribution $\hat{P}(\hat{\tau})$ of first return times is thus a "narrow" distribution, with a finite average value given by [4,11]

$$\langle\hat{\tau}\rangle \simeq \tau_0(v_{\max}/v_{\text{trap}}), \qquad (2)$$

where $v_{\max} > v_R$ is the typical atomic velocity during an escape period ($v_{\max} \approx 2v_R$). The total escape time $\hat{T}(N) = \sum_{i=1}^{N}\hat{\tau}_i$ thus grows as $N\langle\hat{\tau}\rangle$ for $N \gg 1$. The power law dependence on N of $T(N)$ and $\hat{T}(N)$ then gives a simple condition for the filling of the trapping zone: If and only if $\alpha \geq 1$, one has $T(N) \propto N^\alpha \gg \hat{T}(N) \propto N$. In such a case all atoms will accumulate in the trapping zone when $\Theta \to \infty$, ensuring an efficient cooling.

Finally, the width δv_Θ of the velocity distribution of cooled atoms after a time Θ can be derived from the Lévy flight approach [4]:

$$\delta v_\Theta \simeq v_0\left(\frac{\tau_0}{\Theta}\right)^{1/\alpha}. \qquad (3)$$

The smaller α, the faster δv_Θ decreases. This result has been established heuristically in [8], by using Eq. (1) and by writing $\Gamma'(\delta v_\Theta)\,\Theta \simeq 1$, which means that atoms with $|v| < \delta v_\Theta$ remain trapped during Θ [12].

In Raman cooling, as the excitation rate is given by the power spectrum of the pulse intensity, α can be varied by using an appropriate pulse shape. The first Raman cooling experiments used Blackman pulses [2,13] with Raman detunings and pulse durations designed to minimize parasitic excitation around $v = 0$. We have measured the excitation spectrum of a Blackman pulse and found that the best power law fit from $v = 0$ up to half the maximum of $\Gamma'(v)$ corresponds to $\alpha \approx 4$. Here, we alter the exponent α in order to improve the cooling achievable by pulses of a given area. In particular, we have investigated the case $\alpha = 2$, which is simply accomplished by using a time square pulse. Its power spectrum is the square of the sinc function, which consists of a central peak and sidelobes, separated by zeros. Around the first zero, which we make to coincide with the Raman resonance condition for $v = 0$, the spectrum is parabolic. Such pulses are simpler to implement than Blackman pulses. They lead to smaller values of δv_Θ according to Eq. (3), while still ensuring efficient cooling since $\alpha \geq 1$. Moreover, they allow a better use of the available time Θ: For the same spectral width, they are a factor of 2 shorter than a Blackman pulse, so that the pulse repetition rate is higher.

Our experimental setup is an improved version of our all-diode-laser system described in detail in [13]. Briefly, cesium atoms are first captured in a vapor cell MOT and released in free fall at ~6 μK (half-width $\delta v_i = 5.5\hbar k$) in the $F_g = 3$ ground state. They are subsequently illuminated by square pulses of two horizontal Raman beams having a tunable frequency difference $\omega_1 - \omega_2$ around $\omega_{\text{HFS}} = 9.12$ GHz (the cesium hyperfine splitting), a diameter at $1/\sqrt{e}$ of 4 mm, a power of 70 mW (a factor 3 higher than in [13]), and a detuning of -34 GHz between the virtual upper state and the $6p_{3/2}$ state. If the two Raman beams $(\vec{k}_1, \omega_1, \vec{k}_2, \omega_2)$ are counterpropagating, the excitation rate is velocity selective and atoms with $v \neq 0$ are transferred to $F_g = 4$ with a velocity change of $\hbar(\vec{k}_1 - \vec{k}_2)/M \approx 2\hbar\vec{k}/M$. The resonance condition is $\delta = \omega_1 - \omega_2 - \omega_{\text{HFS}} = 2k(v_R + v)$. The sign of the frequency offset δ is chosen such that the velocity change for the resonant atoms is opposed to their velocity. After each Raman pulse, a colinear, 30 μs long, resonant repumping pulse, tuned to $F_g = 4 \to F_e = 4$, excites all $F_g = 4$ atoms and gives them a chance to fall in $F_g = 3$ with $v \approx 0$ after spontaneous emission. A cooling sequence typically takes $\Theta = 20$ ms (limited by gravity and beam size), and the velocity distribution of atoms in $F_g = 3$ is probed using a low power, 3 ms long Blackman pulse providing a velocity resolution of $v_R/20$ or 175 μm /s.

In order to investigate this new cooling strategy, we start with the simplest scheme, consisting of a single 30 μs square Raman pulse, short enough to cover a large fraction of the initial velocity distribution and centered alternately at $v = \pm 4v_R$ [Fig. 2(a), dashed line], the sign change being accomplished by exchanging the directions of the two Raman beams. The peak excitation probability is $\approx$0.4. The sequence *Raman excitation–repumping pulse* at $v = +4v_R$, then at $v = -4v_R$, is repeated 136 times, leading to a remarkable cooling efficiency: 70% of

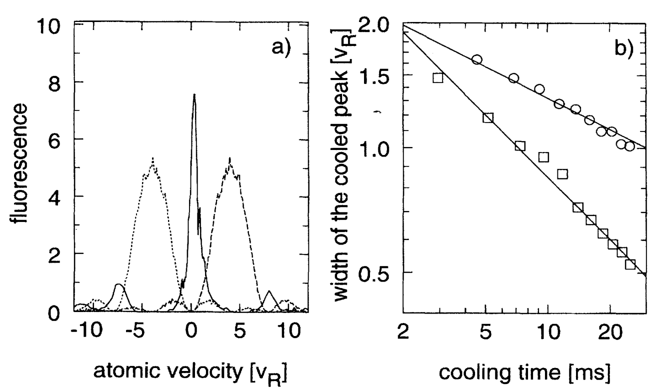

FIG. 2. (a) Dashed line: excitation profile of 30 μs pulses tuned to $\pm 4v_R$. (This profile has been measured independently by using copropagating pulses.) Solid line: velocity distribution after 136 repetitions of these pulses (total time 22 ms). The velocity spread (half-width at $1/\sqrt{e}$) is $0.34v_R$, corresponding to $T_{\text{eff}} \approx 25$ nK. The fluorescence is in units of the maximum signal obtained from the initial distribution. (b) Time dependence of the width of the cooled peak obtained with square pulses (squares) and Blackman pulses (circles). Straight lines are fits by power laws $\Theta^{-1/2}$ and $\Theta^{-1/4}$, respectively.

the atoms are compressed in a peak 7.5 times higher than the initial distribution [Fig. 2(a)]. The velocity spread (half-width at $1/\sqrt{e}$) is $0.34v_R$, much narrower than the width ($\approx 4v_R$) of the hole in $\Gamma'(v)$. This confirms that δv_Θ can be much smaller than v_0, as already indicated by Eq. (3). The corresponding temperature $T_{\rm eff} = T_R/8 =$ 25 nK is equivalent to our previous result using sequences of eight Blackman pulses [13], but with a better fraction of atoms in the cold peak and a considerably simpler pulse sequence. Moreover, the time evolution of the velocity spread obeys very well the $\Theta^{-1/2}$ law of Eq. (3) [Fig. 2(b)]. When the square pulses are replaced by Blackman pulses, centered at the same frequency and twice as long as the square pulses to maintain the same characteristic width, higher temperatures and lower peak heights are obtained for all interaction times [Fig. 2(b)]. In this case, the evolution of the velocity spread is well fitted by a $\Theta^{-1/4}$ law, as predicted by Eq. (3).

In order to lower further the temperature, one would like to decrease the width v_0 of the hole in $\Gamma'(v)$ since this produces a colder final distribution [Eq. (3)]. However, when v_0 is decreased, the pulse is longer in time, it interacts with a smaller fraction of the velocity distribution, and it ultimately gives fewer chances for a given atom to fall near $v = 0$ by spontaneous emission. Furthermore, atoms can also accumulate in the outermost zeros of the excitation profile as can be seen already near $\pm 8v_R$ in Fig. 2(a). The immediate solution to this problem is to use a cooling sequence made of two pulses: a long pulse for good *filtering* (v_0 small) and a short pulse covering a wide velocity range, in order to *recycle* those atoms which do not interact with the narrow pulse. As the recycling pulse is short in duration, its contribution to the total cooling time is small. Raman cooling with two square pulses is shown in Fig. 3(a). The sequence consists of a 120 μs pulse centered at $v \approx \pm v_R$ and a 30 μs pulse at $v \approx \pm 4v_R$ and is repeated 26 times. The cold peak has a $1/\sqrt{e}$ velocity spread of $(0.12 \pm 0.01)v_R$, corresponding to $T_{\rm eff} = 2.8 \pm 0.5$ nK ($T_R/73$), nearly a factor of 10 lower than in the one-pulse cooling scheme. The peak is 10 times as high as the initial distribution and contains 35% of the total number of atoms. Narrower filtering pulses lead to still lower temperatures, but with a reduced gain at $v = 0$. For instance, $T_{\rm eff} = 0.8$ nK $= T_R/250$ has been observed with a peak height increase of 6.4, using a filtering pulse of 400 μs duration.

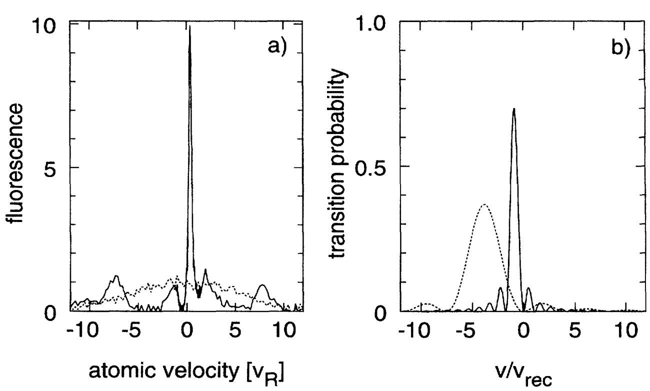

FIG. 3. (a) Raman cooling with 30 μs pulses centered at $\pm 4v_R$ and 120 μs pulses centered at $\pm v_R$, 26 repetitions. The central peak has a $1/\sqrt{e}$ half-width of $0.12v_R$ (an 8% contribution due to the probe linewidth has been subtracted by deconvolution). This corresponds to an effective temperature of $T_{\rm eff} = 2.8$ nK. The dashed curve is the velocity distribution without Raman cooling, corresponding to $T_{\rm eff} \approx 6.0$ μK. (b) The Raman pulses used for the above result (for clarity, only the pulses on the negative-velocity side are shown). As the available power limits the peak excitation probability of the 30 μs pulse to about 0.4, it is repeated 3 times before the 120 μs pulse is applied.

In the search of an optimum square pulse cooling configuration, a compromise must be made between the fraction of cooled atoms and the width of the cooled distribution. We present here a simplified derivation of the width $\delta v_{\Theta,\rm opt}$ of the narrowest peak that can be filled significantly in a given time Θ, i.e., that accumulates $\simeq 50\%$ of the atoms. A rigorous calculation optimizing the height of the peak at $v = 0$ gives the same results, within prefactors of order 1 [14]. The single important parameter of the pulse sequence is the duration θ_0 of the filtering pulse (the narrowest pulse). The excitation at $|v| \geq v_0$ is realized efficiently by much shorter recycling pulses, whose durations are neglected here. θ_0 is related to the width v_0 of the dip in $\Gamma'(v)$ by the condition of the first zero of the sinc function in $v = 0$ for Γ', which implies $\pi/\theta_0 = kv_0$. Adding pulses on both sides of $v = 0$ leads to a sequence duration of $\tau_0 = 2\theta_0$ and the excitation rate becomes $\Gamma'(v \simeq 0) = (v/v_0)^2/2\theta_0$ and $\Gamma'(|v| > v_0) = 1/(2\theta_0)$. The optimization is now done very simply: The narrowest peak that can be filled is defined by a filling time just equal to the total time. The filling time being on the order of the first return time, one has

$$\langle \hat{\tau}(v_{\rm trap} = \delta v_{\Theta,\rm opt}) \rangle \simeq \Theta \, . \tag{4}$$

The problem is now fully characterized by the two equations (3), (4), having two parameters (Θ, $v_{\rm max}$) and two unknowns ($v_{0,\rm opt}$ or equivalently $\theta_{0,\rm opt}$, $\delta v_{\Theta,\rm opt}$). Simple algebra gives the optimum values

$$\frac{v_{0,\rm opt}}{v_R} \simeq \left(\frac{2v_{\rm max}}{v_R}\right)^{2/3} \left(\frac{\pi \tau_R}{2\Theta}\right)^{1/3}, \tag{5}$$

where $\tau_R = 2M/\hbar k^2$ is the recoil time, and

$$\frac{\delta v_{\Theta,\rm opt}}{v_R} \simeq \left(\frac{2v_{\rm max}}{v_R}\right)^{1/3} \left(\frac{\pi \tau_R}{2\Theta}\right)^{2/3}. \tag{6}$$

Note that $\delta v_{\Theta,\rm opt}$ decreases as $\Theta^{-2/3}$, whereas δv_Θ varies only as $\Theta^{-1/2}$ for a fixed v_0 which is not optimized for each value of Θ. For $v_{\rm max} = 2v_R$, $\Theta = 10$ ms $= 130\tau_R$ (the effective total time of the cooling pulses in the experiment) the result is $v_{0,\rm opt} \approx 0.6v_R$ and $\delta v_{\Theta,\rm opt} \approx$

$0.08v_R$, which is 40% narrower than our experimental result. Considering the simplicity of our model and the laser power limitation preventing us from realizing extremely short recycling and repumping pulses with unity transfer efficiency, the agreement is satisfactory. We also performed numerical simulations assuming ideal experimental conditions and $\Theta = 20$ ms $= 260\tau_R$. They predict an optimum corresponding to $T = T_R/400 =$ 0.5 nK and $v_{0,\text{opt}} = 0.4v_R$. For an initial half-width $\delta v_i = 5.5v_R$ the predicted peak height increase is 75.

The Lévy flight analysis can easily be extended to higher dimension d. The distribution of trapping times $P(\tau)$ varies as $P(\tau) \sim 1/\tau^{1+d/\alpha}$ and the first return time is proportional to $(v_{\max}/v_{\text{trap}})^d$. The optimization of Raman cooling can be done along the same lines as above. This is important for the case $d = 3$ since subrecoil Raman cooling has not yet been demonstrated in this case. Preliminary calculations show that the square pulse sequence could lead to a subrecoil temperature, but with only a small fraction of the atoms in the cooled peak. This is due to the fact that in 3D the distribution $P(\tau)$ is broad only if $\alpha \geq d = 3$. Only in this case, the total trapping time $T(N) \propto N^{\alpha/d}$ dominates the total escape time $\hat{T}(N) \propto N$ so that all atoms accumulate in the narrow peak. Hence in 3D, Lévy flights predict more efficient cooling with triangular or Blackman pulses ($\alpha = 4$). It would be interesting to determine the optimum cooling for arbitrary values of α and to extend the approach to the case of trapped atoms which offer attractively long interaction times.

In conclusion, we have pointed out the crucial role of the exponent α and the interaction time Θ in subrecoil cooling. Using Lévy statistics, we have predicted and demonstrated experimentally that in 1D square pulses ($\alpha = 2$) are not only simpler to implement but also lead to a faster cooling ($\delta v_\Theta \propto \Theta^{-1/2}$) than Blackman pulses ($\alpha = 4$ and $\delta v_\Theta \propto \Theta^{-1/4}$). For cesium atoms we have obtained effective temperatures below 3 nK or $T_R/70$, which are to our knowledge the lowest temperatures achieved in 1D laser cooling. Our analysis also provides analytical expressions for the optimal cooling in a given time Θ. Finally, in higher dimension d, we predict that only if $\alpha \geq d$ all atoms accumulate in the subrecoil peak.

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[11] This expression is similar to the equilibrium time calculated in [2].

[12] The heuristic derivation reduces the cooling process to a pure filtering during Θ. Actually the atoms can leave the trapping zone and come back several times during Θ. Several filtering periods corresponding to the various trapping times $\tau_1, \tau_2, \ldots$ should be taken into account and the evaluation of the width of the velocity distribution is nontrivial. It is only because the sum $T(N)$ is dominated by a very small number of terms, a typical property of Lévy sums [10], that Eq. (3) is valid.

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